Regular Orbits and Periodic Loops in Multiply–Barred Galactic Potentials

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ABSTRACT

We show that non-chaotic multiply-periodic particle orbits can exist in a galaxy-like potential where a small fast-tumbling nuclear bar is nested inside a main bar which has a slower pattern speed. We introduce the concept of a *loop:* a one-dimensional curve such that particles distributed along it at some initial instant return to the same curve (as viewed in the rotating frame of one of the bars) after the bars return to the same relative position. Just as particles following regular orbits in a simply-barred potential are trapped around closed periodic orbits, so regular orbits in a doubly-barred potential oscillate about stable loops. We find both loops which remain aligned with the inner bar and loops aligned with the outer bar: particles trapped around these loops could form the building blocks for a long-lived, self-consistent, doubly-barred galaxy. In a realistic doubly-barred galaxy potential, we demonstrate the existence of stable loops which support the structure of the inner bar. We use the epicyclic approximation to preview the pattern of gas flow in a doubly-barred potential.

Subject headings: Stellar Dynamics — Galaxies: Active — Galaxies: Kinematics and Dynamics — Galaxies: Structure

1. INTRODUCTION

The idea of 'bars-within-bars', with a small nuclear bar lying inside a slower-rotating kiloparsec-scale bar, was introduced to explain how gas could flow into the center of a galaxy with an active nucleus, to feed 'the monster' lurking there (*e.g.* Shlosman, Frank & Begelman 1989). Torques due to a single bar can efficiently remove angular momentum from gas and cause it to move inwards; but if the bar has an inner Lindblad resonance, the gas accumulates in this region, which in a kiloparsec-scale bar is generally at least 100 pc out, and flow into the center is slow (*e.g.* Piner *et al.* 1995). Nuclear bars nested within the main bar and having a whole range of position angles relative to it have been observed in the optical and near IR (see Shaw *et al.* 1993; 1995; Wozniak *et al.* 1995; Friedli 1996; Erwin *et al.* 1997). Self-consistent N-body simulations including a gas component (Friedli & Martinet 1993; Heller & Shlosman 1994; Combes 1994) have produced double bars which persist for at least several rotation periods and significantly influence the gas distribution in the inner parts of the galaxy.

Because there is no reference frame in which the potential of a doubly-barred system is constant, the particle motions are often assumed to be chaotic: we show here that this is a misconception; these potentials admit families of regular, multiply-periodic orbits. Our work runs somewhat parallel to that of Louis & Gerhard (1988) and of Sridhar (1989), who studied orbits in a pulsating spherical system.

2. THE CONCEPT OF A LOOP

In the symmetry plane of a single steadily-rotating bar, there are two principal families of stable orbits which close in the rotating frame of the bar, and lie within the corotation radius, at which the pattern speed of the rotating bar is equal to the orbital period of the disk stars: they are the x_1 and x_2 orbits, elongated respectively parallel and perpendicular to the bar. In a self-consistent bar, most of the stars will be on orbits trapped around the x_1 family (*e.g.* Contopoulos & Papayannopoulos 1980, Sparke & Sellwood 1987). On these periodic orbits, only the action associated with a star's orbit around the center is non-zero, and the motion is simply periodic.

Choosing spherical polar coordinates R, θ in the plane of the barred disk, we can approximate the potential $\Phi(R, \theta)$ of a weakly barred galaxy as the sum of an axisymmetric part $\Phi_0(R)$ and a bisymmetric term

$$\Phi_1(R, \theta) = \Phi_B(R) \cos[2(\theta - \Omega_B t)], \tag{1}$$

where Ω_B is the bar's angular velocity. The motion of a test particle in this potential can be decomposed into the 'guiding center' motion, on a circle of radius R_0 with the angular velocity $\Omega(R_0)$ corresponding to the axisymmetric potential Φ_0, together with the epicyclic oscillations resulting from the forcing term Φ_1, and a free oscillation at the local epicyclic frequency $\kappa(R_0)$. On a periodic orbit, this last oscillation is absent, and in the rotating frame of the bar we can write the deviations R_1, θ_1 from a circular orbit as

$$R_1 = B_R \cos(\omega t + 2\theta_S); \qquad \theta_1 = B_\theta \sin(\omega t + 2\theta_S), \tag{2}$$

where the frequency $\omega = 2[\Omega(R_0) - \Omega_B]$, and the constants B_R, B_θ are functions of the potential; θ_S gives the initial position of the particle (see *e.g.* Section 3.3.3 of Binney & Tremaine 1987). Since the particle returns to the same point after a time $2\pi/\omega$, this orbit is closed.

A second smaller bar would correspond to an additional potential of the form (1), where Φ_B and Ω_B are replaced by the quantities Φ_S and Ω_S appropriate to the second bar. Since the equations of motion are linear, the new forced oscillation is the sum of those induced by the two bar potentials separately; in the frame rotating with the larger bar, we can write the displacement in radius as

$$R_1 = B_R \cos[\omega t + 2\theta_S] + S_R \cos[(\omega - \omega_p)t + 2\theta_S], \tag{3}$$

where $\omega_p = 2(\Omega_S - \Omega_B)$, with a similar equation for θ_1. Here θ_S is the initial angular coordinate of the particle, where we measure the time t from the moment at which the two bars are aligned. *We define a loop as the one-dimensional curve defined by varying θ_S: if at any instant one places particles along this curve, then after a time $2\pi/\omega_p$, they will all have returned to the curve, but will have moved around it by an angle $2\pi\omega/\omega_p$.* In this linear approximation, a loop is a set of particles which have the same guiding radius, and lack any free epicyclic motion, but respond only to the periodic forcing of the two bars; a general orbit with that same guiding center will oscillate about the loop.

Abandoning the epicyclic approximation, the two-dimensional problem of motion in the symmetry plane of a general doubly-barred system can be reformulated as that of motion in a three-dimensional time-steady potential (Lichtenberg & Lieberman 1992, p.15). A regular orbit in such a potential can be expressed as the sum of motions with three independent frequencies, which are the relative pattern speed ω_p, and two others which correspond in the epicyclic limit to ω and κ; on a loop, the action corresponding to the last of these, the free radial oscillation, is zero. In this case, we must find the loops numerically. Clearly, the loop concept can be extended to a potential with 3 or more periodically varying components.

3. LOOPS IN A REALISTIC GALAXY POTENTIAL

As an example of a realistic double-barred galaxy potential, we modified the 'standard' single-bar model used by Athanassoula (1992) and Piner et $al.$ (1995), consisting of an axisymmetric disk and bulge, and an $n = 2$ Ferrers bar. The larger bar in our model is prolate with axial ratio $b/a = 0.4$, the quadrupole moment is $Q_m = 2.25 \times 10^{10}$ M$_\odot$ kpc^2 and its semimajor axis is $a_B = 7$ kpc; the radius of the Lagrangian point L_1 is $r_L = 1.2 a_B$. The small bar is also an $n = 2$ Ferrers bar, with its mass, linear size, and Lagrangian radius all set at 60% of the corresponding quantities for the big bar. The central density and core radius of the bulge are adjusted so that the central density of the whole system is 1.0×10^{10} M$_\odot$ kpc^{-2} and the mass within 10 kpc is 5×10^{10} M$_\odot$. Our potential has a somewhat weaker bar than the standard model of Athanassoula and Piner et $al.$, with half the quadrupole moment, because we want to compare our exact loops with those predicted by the epicycle approximation.

In the epicyclic approximation, our small bar has inner Lindblad resonances at 0.6 and 1.3 kpc; its corotation radius is at 4.9 kpc, beyond the outer of the inner Lindblad resonances of the larger bar at 2.9 kpc. We solve the linearized equations of motion in the $\cos(2\theta)$ component of this potential, to find the loops in the middle column of Figure 1. The four innermost loops behave like the inner x_1 orbits in the potential of the smaller bar alone; the inner two are almost circular, and the next two follow the small bar as it rotates inside the larger bar. Particles following orbits trapped around these loops should be able to support the figure of the small bar. Further out, as the influence of the large bar becomes stronger, the loops change smoothly from approximately following the x_1 orbits in the small bar, to being the counterparts of the x_2 orbits in the big bar. Beyond the inner Lindblad resonance of the large bar at radius 2.9 kpc, the loops are elongated along the X axis and behave like x_1 orbits in the large bar. The small bar has little influence at these radii, except close to its corotation point at $R = 4.9$ kpc. The epicyclic approach fails close to the Lindblad resonances and the corotation point of either bar, since the perturbations R_1 and θ_1 become large.

When we return to the nonlinear equations of motion in the exact potential, we can again seek one-dimensional closed curves which transform into themselves after a time $2\pi/\omega_p$, when the two bars return to the same relative position. In general, the frequency ω_p of the two bar patterns relative to each other is not commensurable with the frequency ω with which any individual particle circumnavigates the loop; so the position and velocity of a particle on a generic loop at successive times $2N\pi/\omega_p$, where $N = 1, 2, \ldots$, will fill the one-dimensional curve which is the loop. In the rotating frame in which the large bar remains parallel to the X axis, we searched for loops by following a particle that starts on the Y axis at an instant when the bars are aligned. We restricted ourselves to symmetric loops, for which the starting velocity along the Y direction is zero, and varied the starting X velocity so as to converge on the loop, beginning from an initial estimate given by the epicyclic approximation. The left column of Figure 1 shows one of our loop solutions, with the epicyclic approximation for comparison. The dots show successive positions of a particle launched near the loop and followed for 400 orbits; it stays close to the loop, which is seen to be stable. Although (just as in a single bar) particle orbits become chaotic near strong resonances, loops exist over most of the range in radius, suggesting that a self-consistent double bar could be built from particles on orbits trapped around them.

The central column of Figure 1 shows that many of the loops in our doubly-barred potential which lie within the small bar (semimajor axis $a_S = 4.2$ kpc) remain aligned with it; particles following orbits trapped around these loops can support the figure of this bar. If the small bar rotated somewhat more rapidly, its corotation radius would coincide with large bar's inner Lindblad resonance; Tagger et $al.$ (1987)

have suggested that a secondary bar is most likely to form with resonances in this relation. In that case, loops which are confined within the small bar would not be influenced by any of the resonances of the big bar, and almost all of them would remain aligned with the small bar as it rotates relative to the larger bar; this would be the most favorable case for building a strong secondary bar. Louis & Gerhard (1988) have shown that two bars which make up a self-consistent double-bar system must distort slightly as they rotate through each other; but since this deformation has the same periodicity as the relative rotation of the bars, it will not introduce any extra frequencies into the potential, and the loops should remain qualitatively similar to those we calculate here.

4. GAS FLOW IN DOUBLE BARS

Lindblad & Lindblad (1994), and Wada (1994), added a dissipative term to the equations of motion in a single bar, linearized the equations using the epicyclic approximation, and studied the closed periodic orbits in this system. With the dissipative term included, the deviations from circular orbits are finite at the Lindblad resonances, and the major axis of the closed orbits rotates smoothly through 90 degrees; but the epicyclic approximation still breaks down at corotation. Inside corotation, a surprising correspondence was found: the pattern of closed orbits closely reflects the gas flows modeled hydrodynamically — the damped epicyclic equations give us a cheap way to preview the hydrodynamic simulation. These two papers used slightly different versions of the frictional term, and their results are substantially independent of the exact form chosen.

Unlike stars, gas clouds orbiting in a single bar potential can only fill orbits which do not cross themselves or intersect each other: gas clouds on self-crossing or intersecting orbits would collide, removing the gas from these orbits. In a double bar, if loops which are solutions of linearized equations of motion including dissipation should cross themselves or intersect other loops *at any given time*, gas will be quickly swept out of these loops. In regions where orbits or loops crowd together, shocks in gas are expected, which would be visible in a real galaxy as dark dust lanes.

In order to preview gas flows in a doubly-barred potential, we included in our equations of motion a dissipative term

$$\mathbf{F}_{\text{friction}} = -2\lambda \left[\ \dot{\mathbf{r}} \ - \ \mathbf{\Omega}(R) \times \mathbf{r} \ \right] /R \tag{4}$$

proportional to the difference between the particle's velocity and the speed on a circular orbit at that radius; the constant λ prescribes the strength of the friction. The results for $\lambda=16$ km s^{-1}, which at $R=4$ kpc is equal to the value used by Lindblad & Lindblad, are presented in the right column of Figure 1. This force corresponds only to about 8% of the circular velocity on the flat part of the rotation curve in our potential, so it is small in relation to other speeds in the problem.

The flow breaks into two parts: the outer pattern is generated by the big bar and is almost steady in the rotating frame, while the inner one rotates along with the smaller bar. The shocks induced separately by each bar are at times connected (the two lower panels on Fig.1) while at other relative phases of the two bars, they are disconnected (two upper panels). Connected shocks may speed up the gas inflow into the galaxy center, which might give rise to a long-timescale periodicity in the feeding of an active nucleus. In the dissipative epicyclic approximation, particles remain on closed orbits or periodic loops, so we cannot use this scheme to estimate the rate of gas inflow. Depending on the relative location of resonances induced by both bars, the loop patterns can interfere with each other in different ways: so the arrangement of dust lanes or the distribution of gas can be used as a diagnostic of the bar pattern speeds in observed double-bar

systems.

REFERENCES

Athanassoula, E. 1992, MNRAS, 259, 328

Binney J., & Tremaine S. 1987, Galactic Dynamics, (Princeton: Princeton Univ. Press)

Combes, F. 1994, in Mass-Transfer Induced Activity in Galaxies, ed. I. Shlosman, (Cambridge: Cambridge University Press), 170

Contopoulos, G., & Papayannopoulos, Th. 1980, A&A, 92, 33

Erwin, P., Sparke, L.S., & Gallagher, J.S. 1996, BAAS, 28, 837

Friedli, D. 1996, in ASP Conf. Proc. 91, Barred Galaxies, IAU Colloquium 157, ed. R. Buta *et al.* , (San Francisco: ASP), 378

Friedli, D., & Martinet, L. 1993, A&A, 277, 27

Heller, C.H, & Shlosman, I. 1994, ApJ, 424, 84

Lichtenberg, A.J., & Lieberman, M.A. 1992, Regular and Chaotic Dynamics, 2nd edition, (New York: Springer)

Lindblad, P.O., & Lindblad, P.A.B. 1994, in ASP Conf. Proc. 66, Physics of the Gaseous and Stellar Disks of the Galaxy, ed. I.R. King, (San Francisco: ASP), 29

Louis, P.D., & Gerhard, O.E. 1988, MNRAS, 233, 337

Piner, B.G., Stone, J.M., & Teuben, P.J. 1995, ApJ, 449, 508

Shaw M.A., Combes, F., Axon, D.J., & Wright, G.S. 1993, A&A, 273, 31

Shaw M.A., Axon, D., Probst, R., & Gatley, I. 1995, MNRAS, 274, 369

Shlosman, I., Frank, J., & Begelman, M.C. 1989, Nature, 338, 45

Sparke, L.S., & Sellwood, J.A. 1987, MNRAS, 225, 653

Sridhar, S. 1989, MNRAS, 238, 1159

Tagger, M., Sygnet, J.F., Athanassoula, E., & Pellat, R. 1987, ApJ, 318, L43

Wada, K. 1994, PASJ, 46, 165

Wozniak, H., Friedli, D., Martinet, L., Martin, P., & Bratschi, P. 1995, A&AS, 111, 115

FIGURE CAPTIONS:

Figure 1: Loops (sets of particles) followed for 4 relative positions of the bars, in the rotating frame of the large bar, which lies along the X-axis. *Left:* Exact loops (solid lines) remain close to the epicyclic

approximation (dashed). The two crossed lines mark the orientation of the large and the small bar and extend to the outer of their inner Lindblad resonances. The dots represent successive positions of a generic particle that started close to the loop, and remains trapped about it. *Center:* In the same potential, loops calculated in the epicyclic approximation (solid lines) for the same set of bar orientations. Dashed circles, from the outermost inwards, mark corotation of the small bar at 4.9 kpc, the outer inner Lindblad resonance of the large bar at 2.9 kpc, and the two inner Lindblad resonances of the small bar at 1.3 and 0.6 kpc. *Right:* In the same potential, and for the same bar angles, loops for the 'dissipative epicyclic' equations, with the dissipative coefficient $\lambda = 16$ km s^{-1}.